Brief Announcement: Maintaining a Bounded Degree Expander in Dynamic Peer-to-Peer Networks

Aalto University
School of Science

Antonio Cruciani

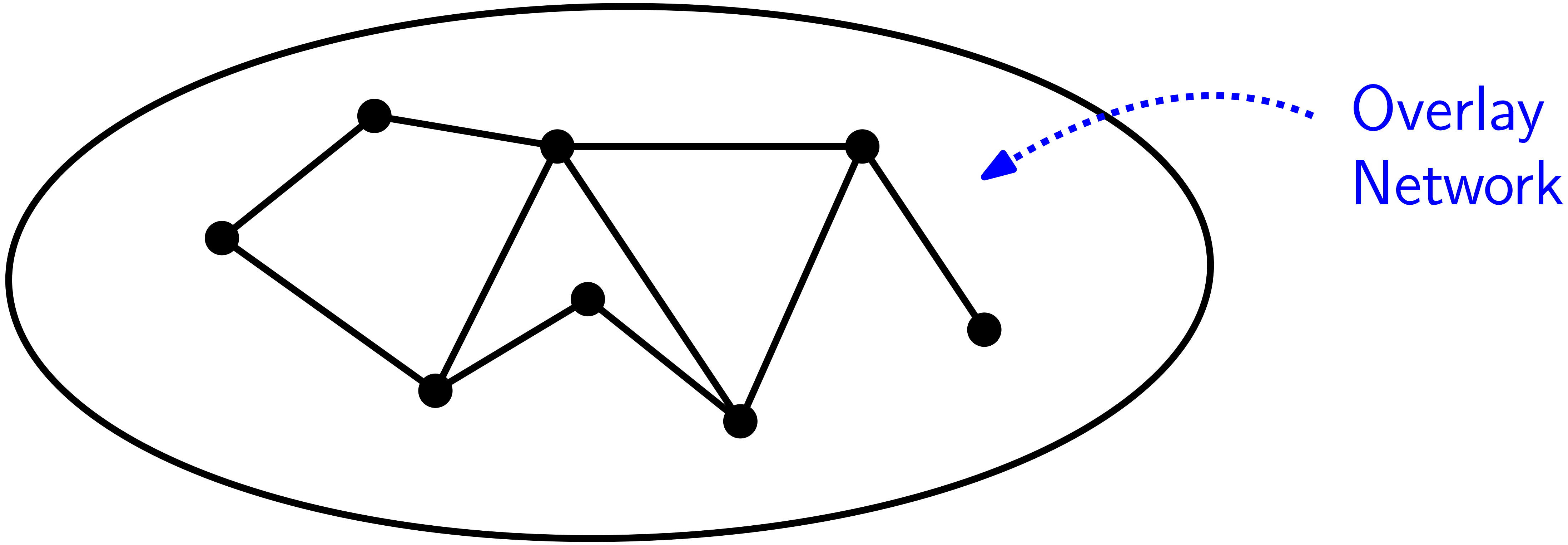
antonio.cruciani@aalto.fi

Peer to Peer Networks

Prevailing Definition: A network of peers, ideally fully decentralized

Key Challenges:

- Highly dynamic
- High **churn**



Underlying Internet (Complete Connectivity)

Model: Dynamic Network with Churn (DNC)

Synchronous: All nodes follows the same clock. In each round $r = 1, 2, 3, \ldots$

- each node that joined the network has access to the uniform distribution over V_r

Adversarial Dynamism:

An **oblivious** adversary (knows the algorithm but not the coin toss outcomes) designs the churn

$$\mathcal{G} = \left(G^0, G^1, \ldots, G^r, \ldots\right)$$

- Must "attach" each joining node at time r with at least one pre-existing node.
- Keep the degree $\mathtt{deg}(G^r) \leq \delta$

Bootstrap and Maintenance Phases

$\mathcal{O}(\log n)$ rounds Bootstrap Phase

Algorithm initialization

Adversary wakes up

Churn rate of up to $\mathcal{O}(n/\text{polylog}(n))$ per round

Maintenance Phase

We need to cope with the churn

RAES Protocol

Algorithm 2: Overview of the RAES-style protocol [Becchetti et al., SODA 2020].

Input : Min degree d, Max degree Δ

Let $G = (V, \{\emptyset\})$.

for $t \leftarrow 0, 1, 2, \ldots$ **do**

  Phase 1 (reconnection): Each node u with degree $d(u) < d$ picks $d - d(u)$ new neighbors uniformly at random.

  Phase 2 (degree adjustment): Each node u with degree $d(u) > \Delta$ selects $d(u) - \Delta$ neighbors uniformly at random and drops the connection.

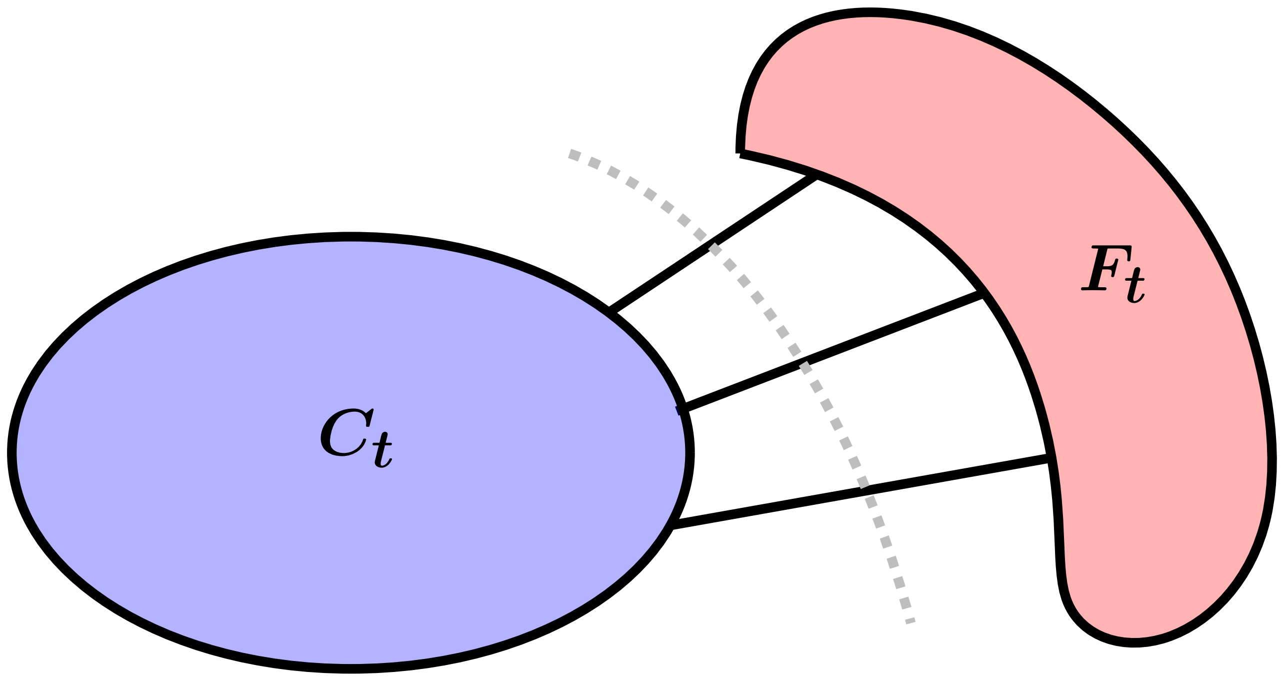
Converges to a bounded degree expander graph in $\mathcal{O}(\log n)$ rounds whp.

Question: What happens if nodes can join and leave the network?

Empirical evidence that can tolerate churn [Cruciani & Pasquale ICDCN'23]

Observation

Problem: There is an adversarial strategy that deteriorates the expansion whp.

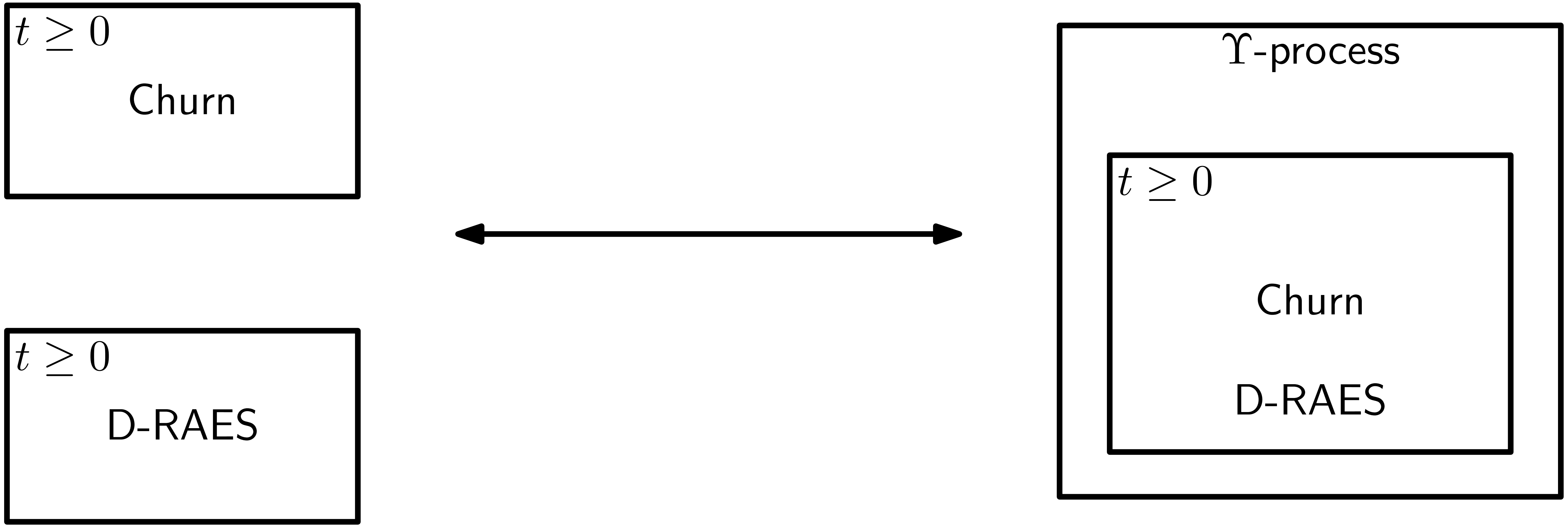


Solution:

Phase 0 (refresh neighbors): With probability $1/\text{polylog}(n)$ each node u with degree $d(u) \in [d, \Delta]$ drops all its neighbors.

Overview of the results

We adapt the techniques by Augustine et al. [FOCS 2015] to our problem



Theorem 1 *The D-RAES protocol maintains a dynamic graph $(G_1, \ldots, G_t, \ldots)$ such that, with high probability, each snapshot contains a large $n - o(n)$-sized expander in which each node has degree in $[d, \Delta]$ despite a churn rate of $\mathcal{O}(n/\log^k n)$ for $k \geq 1$.*

Thank You

The D-RAES

Algorithm 1: Overview of the D-RAES.

Input : Min degree d, Max degree Δ

Let $G = (V, \{\emptyset\})$.

Bootstrap phase (no churn, requires B rounds)

for $t \leftarrow B+1, B+2, \ldots$ **do**

- **Phase 0 (refresh neighbors):** With probability $1/\text{polylog}(n)$ each node u with degree $d(u) \in [d, \Delta]$ drops all its neighbors.
- **Phase 1 (reconnection):** Each node u with degree $d(u) < d$ picks $d - d(u)$ new neighbors uniformly at random.
- **Phase 2 (degree adjustment):** Each node u with degree $d(u) > \Delta$ selects $d(u) - \Delta$ neighbors uniformly at random and drops the connection.